



## **Study proves the existence and solves the mystery of the rotating Kaleidocycles**

Researchers derive exact formulae to show that rotating Kaleidocycles can always be built from six or more hinged tetrahedra

**Fukuoka, Japan**— Kaleidocycles are flexible polyhedral structures composed of rigid tetrahedra connected along their edges to form rotating rings. Each tetrahedron is a solid 3D polygon with four triangular faces (like a triangular pyramid), and the hinges connect neighboring units, enabling a smooth rotational motion of the ring without deforming the individual pieces. These mechanisms are often compared with the bubble rings blown by dolphins.

Although Kaleidocycles have been known for more than 50 years and are widely appreciated as origami curiosities, no study has rigorously proven their existence for general ring sizes or provided exact formulae describing how they move. Also, designing such continuously rotating Kaleidocycles is difficult because many linked systems jam, wobble, or move unpredictably.

In the present study, published in the journal [\*Studies in Applied Mathematics\*](#) on May 13, 2026, [Assistant Professor Shota Shigetomi](#) and [Director Kenji Kajiwara](#) from Kyushu University's [Institute of Mathematics for Industry](#), together with Professor Shizuo Kaji from Kyoto University, Japan, transformed the linkage problem of Kaleidocycles into a geometric one.

Instead of analyzing the hinges directly, they represented the mechanism as a discrete spatial curve with a constant twisting angle. The researchers then applied elliptic theta functions, a class of functions used to describe repeating patterns, to build explicit formulae for periodic motion and rotating ring closure.

“We were inspired by the intriguing properties of origami, which can connect seemingly distant areas of mathematics through a tangible object,” says Shigetomi. “This study has brought together researchers from fields such as geometry, topology, and integrable systems (exactly solvable systems).”

The researchers developed explicit formulae for constructing Kaleidocycles and showed that they can be constructed whenever six or more identical tetrahedra are linked in a ring. Earlier classical examples focused mainly on six-unit rings, while broader cases lacked rigorous confirmation. The team also showed that their developed formulae satisfy well-known nonlinear motion equations called the modified KdV and sine-Gordon equations, and that the trajectories of the curve motion form beautiful geometric shapes known as semi-discrete constant negative curvature surfaces (semi-discrete K-surfaces). These findings reveal that the motion of the mechanism can be studied using tools originally developed in mathematical physics, while also uncovering deep connections with discrete differential geometry.

Numerical analysis in this work suggested that the constructed Kaleidocycles possess a single degree of freedom, meaning they may move in a single, controlled, and efficient way. Moreover, the authors note that more general linkage mechanisms remain difficult to analyze,

and a formal proof that these Kaleidocycles always have exactly one degree of freedom is still needed.

“Our work highlights how multiple areas of modern mathematics are connected through origami. It is also a powerful way to communicate the beauty of mathematics, especially to younger audiences,” adds Shigetomi.

Overall, this study provides a firm mathematical basis for understanding a famous moving origami mechanism and offers methods that may help future designers evaluate controllable linkages for stirring systems, deployable antennas, molecular robots, and related devices.

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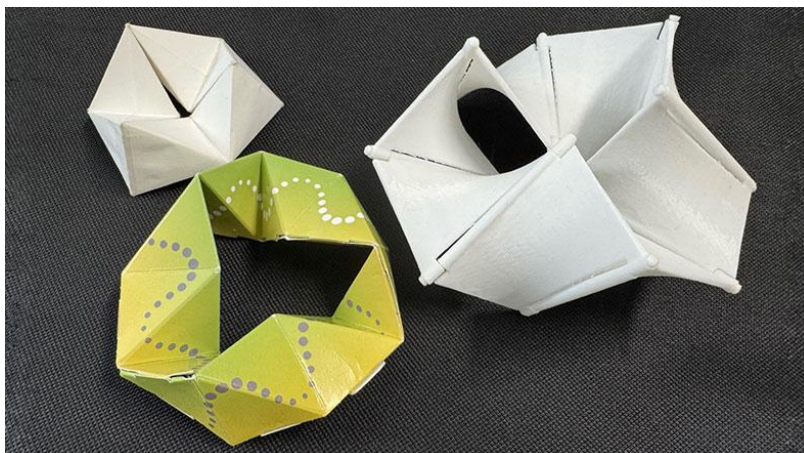
For more information about this research, see "An Explicit Construction of Kaleidocycles by Elliptic Theta Functions," Shizuo Kaji, Kenji Kajiwara, and Shota Shigetomi, *Studies in Applied Mathematics*, <https://doi.org/10.1111/sapm.70224>.

Introductory video of a Möbius Kaleidocycle: <https://youtu.be/NULt0InuVFU>

Origami Diagrams and 3D Models: <https://github.com/shizuo-kaji/Kaleidocycle/blob/master/README.md>

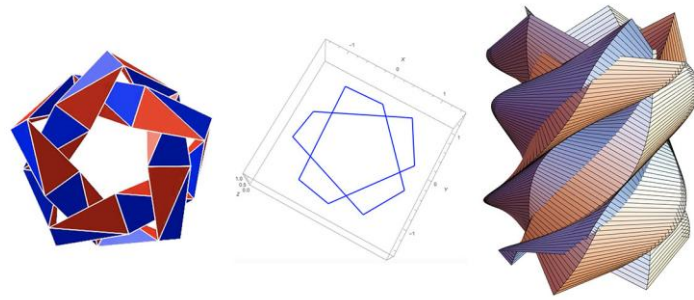
### About Kyushu University

Founded in 1911, [Kyushu University](#) is one of Japan's leading research-oriented institutions of higher education, consistently ranking as one of the top ten Japanese universities in the Times Higher Education World University Rankings and the QS World Rankings. Located in Fukuoka, on the island of Kyushu—the most southwestern of Japan's four main islands—Kyushu U sits in a coastal metropolis frequently ranked among the world's most livable cities and historically known as Japan's gateway to Asia. Its multiple campuses are home to around 19,000 students and 8,000 faculty and staff. Through its [VISION 2030](#), Kyushu U will “drive social change with integrative knowledge.” By fusing the spectrum of knowledge, from the humanities and arts to engineering and medical sciences, Kyushu U will strengthen its research in the key areas of decarbonization, medicine and health, and environment and food, to tackle society's most pressing issues.



**Fig.1** Möbius Kaleidocycles

Continuously rotating Kaleidocycles produced via origami and 3D printing. A physical manifestation of complex concepts in differential geometry, topology, and integrable systems.



**Fig.2** Kaleidocycle, its corresponding discrete curve, and semi-discrete K-surface  
(Left) An example of 15-Kareidocycle. (Middle) 15-Kaleidocycle as a discrete curve. (Right) The semi-discrete K-surface obtained as its trajectory..

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